

QED Corrections to the Scattering of Solar Neutrinos and Electrons^{*}

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Abstract

We discuss recent calculations of the $O(\alpha)$ QED corrections to the recoil electron energy spectrum in neutrino electron scattering, and to the spectrum of the combined energy of the recoil electron and a possible accompanying photon emitted in the scattering process. We then examine the role of these corrections in the interpretation of precise measurements from solar neutrino electron scattering experiments.

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1 Introduction

The calculation of the neutrino electron scattering cross section has a long history. The cross section for the process $\nu_e + e \rightarrow \nu_e + e$ was first computed by Feynman and Gell-Mann almost half a century ago within the framework of an effective four-fermion V–A theory [1]. The QED corrections to this cross section were calculated in 1964 by Lee and Sirlin [2], and shortly afterwards Ram [3] extended their calculations by including hard photon emission. A few years later, 't Hooft computed the lowest order prediction to this differential cross section in the Standard Model (SM) [4]. Since then, the radiative corrections to this process, which plays a fundamental role in the study of electroweak interactions, have been investigated by many authors, focusing on various aspects of the problem [5–11].

't Hooft's early SM predictions were used by Bahcall to examine the total cross section, energy spectrum and angular distribution of recoil electrons resulting from the scattering with solar neutrinos [12]. In a later work, Bahcall, Kamionkowski and Sirlin performed a detailed investigation of the radiative corrections to these recoil electron spectra and total cross sections [13]. Their results showed the importance of these corrections for the analysis of precise solar ν – e scattering experiments, particularly of those measuring the higher energy neutrinos that originate from ^8B decay. In a very recent article [14], we extended previous calculations of the $O(\alpha)$ QED corrections to the recoil electron energy spectrum and evaluated the corresponding corrections to the differential cross section with respect to the total combined energy of the recoil electron and a possible accompanying photon. In the same paper, on which this note is based, we examined the role of these two different radiative corrections in the interpretation of precise measurements from solar neutrino electron scattering experiments.

The energy spectrum of electrons from solar neutrino scattering, first measured by the Kamiokande collaboration [15], provides important information for the investigation of possible solutions to the long standing solar neutrino problem, namely the large deficit in the observed neutrino flux from the Sun with respect to the theoretical predictions [16]. Indeed, while solar neutrino fluxes are predicted by the standard solar model, the shapes of the neutrino energy spectra were shown by Bahcall to be essentially independent of all solar parameters [17]. The shape of the energy spectrum of recoil electrons resulting from the scattering with solar neutrinos can be therefore accurately calculated in a solar model-independent way and compared with the experimental measurements. Distortions of the electron spectrum can be interpreted in terms of neutrino oscillations. In particular, different flavor oscillation solutions to the solar neutrino problem are reflected in characteristic modifications of the shape of the electron spectrum and precise theoretical predictions and experimental measurements are needed to discriminate among them [18, 19]. (See also ref. [20] for a study of the possibility of determining the flavor content of the low-energy solar

neutrino flux based on the analysis of the shapes of the recoil electron spectra.) We refer the reader to ref. [21] for an updated global two- and three-neutrino oscillation analysis of solar neutrino data which includes the electron spectrum measured by the Super-Kamiokande collaboration [22].

In the following we examine the $O(\alpha)$ QED corrections to the SM prediction for neutrino electron scattering, with contributions involving either neutral currents (as in the $\nu_{\mu,\tau} + e \rightarrow \nu_{\mu,\tau} + e$ process) or a combination of neutral and charged currents (as in the $\nu_e + e \rightarrow \nu_e + e$ process). In this analysis we make the approximation of neglecting terms of $O(q^2/M_W^2)$, where q^2 is the squared four-momentum transfer and M_W is the W boson mass. Within this approximation, which is excellent for present experiments ($|q^2/M_W^2| \sim 1$ when the electron recoil energy $\sim 6 \times 10^3$ TeV!), the SM radiative corrections to these processes can be naturally divided into two classes. The first, which we will call “QED” corrections, consist of the photonic radiative corrections that would occur if the theory were a local four-fermion Fermi theory rather than a gauge theory mediated by vector bosons; the second, which we will refer to as the “electroweak” (EW) corrections, will be the remainder. This split-up of the QED corrections is sensible as they form a finite (both infrared and ultraviolet) and gauge-independent subset of diagrams. We refer the reader to ref. [23] for a detailed study of this separation.

The QED radiative corrections are due to both loop diagrams (virtual corrections) and to the bremsstrahlung radiation (real photons) accompanying the scattering process. Of course, only this combination of virtual and real photon corrections is free from infrared divergences. To order α , the bremsstrahlung events correspond to the inelastic process $\nu_l + e \rightarrow \nu_l + e + \gamma$ ($l = e, \mu$ or τ). Experimentally, bremsstrahlung events in which photons are too soft to be detected are counted as contributions to the elastic scattering $\nu_l + e \rightarrow \nu_l + e$. The cross section for these events should be therefore added to the theoretical prediction of the elastic cross section, thus removing its infrared divergence.

We will divide the bremsstrahlung events into “soft” (hereafter SB) and “hard” (hereafter HB), according to the energy of the photon being respectively lower or higher than some specified threshold ϵ . We should warn the reader that the words “soft” and “hard” may be slightly deceiving. Indeed, if ϵ is large (small), the SB (HB) cross section will also include events with relatively high (low) energy photons. While calculations of both soft and hard bremsstrahlung are often performed under the assumption that ϵ is a very small parameter, much smaller than the mass of the electron or its final momentum, we will also discuss results for the case in which ϵ is an arbitrary parameter constrained only by the kinematics of the process. Indeed, the HB cross section (contrary to the SB one) is by itself, at least in principle, a physically measurable quantity for any kinematically allowed value of this threshold. All calculations have been carried out without neglecting the electron mass.

In sect. 2 we present the lowest order prediction for the final electron spectrum,

together with its QED corrections (virtual, soft and hard contributions). In sect. 3 we examine the spectrum of the total combined energy of the recoil electron and a possible accompanying photon emitted in the scattering process. We summarize the main results in sect. 4.

2 QED Corrections to the Electron Spectrum

The SM prediction for the elastic neutrino electron differential cross section is, in lowest-order and neglecting terms of $O(q^2/M_W^2)$ [4],

$$\left[\frac{d\sigma}{dE}\right]_0 = \frac{2mG_\mu^2}{\pi} \left[g_L^2 + g_R^2 (1-z)^2 - g_L g_R \left(\frac{mz}{\nu} \right) \right], \quad (1)$$

where m is the electron mass, $G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant [24], $g_L = \sin^2\theta_w \pm 1/2$ (upper sign for ν_e , lower sign for $\nu_{\mu,\tau}$), $g_R = \sin^2\theta_w$ and $\sin\theta_w$ is the sine of the weak mixing angle. In this elastic process E , the electron recoil energy, ranges from m to $E_{\text{max}} = [m^2 + (2\nu + m)^2]/[2(2\nu + m)]$, $z = (E - m)/\nu$ and ν is the incident neutrino energy in the frame of reference in which the electron is initially at rest. We will refer to the L , R and LR parts of an expression to indicate its terms proportional to g_L^2 , g_R^2 and $g_L g_R$, respectively. For example, the R part of $[d\sigma/dE]_0$ (eq. (1)) is $(2mG_\mu^2/\pi)g_R^2(1-z)^2$.

According to the definition discussed earlier, the one-loop QED corrections to neutrino electron scattering consist of the photonic vertex corrections (together with the diagrams involving the field renormalization of the electrons) computed with the local four-fermion Fermi Lagrangian. These corrections give rise to the following expression for the differential cross section:

$$\left[\frac{d\sigma}{dE}\right]_{\text{Virtual}} = \frac{2mG_\mu^2}{\pi} \left[\frac{\alpha}{\pi} \delta(E, \nu) \right], \quad (2)$$

where

$$\begin{aligned} \delta(E, \nu) = & g_L^2 \left\{ V_1(E) + V_2(E) \left[z - 1 - \frac{mz}{2\nu} \right] \right\} \\ & + g_R^2 \left\{ V_1(E) (1-z)^2 + V_2(E) \left[z - 1 - \frac{mz}{2\nu} \right] \right\} \\ & - g_L g_R \left\{ [V_1(E) - V_2(E)] \left(\frac{mz}{\nu} \right) + 2V_2(E) [z - 1 - z^2] \right\}, \end{aligned} \quad (3)$$

$$V_1(E) = \left(2 \ln \frac{m}{\lambda} \right) \left[1 - \frac{E}{2l} \ln \left(\frac{E+l}{E-l} \right) \right] - 2 - \frac{E}{l} \left[\text{Li}_2 \left(\frac{l-E+m}{2l} \right) \right]$$

$$- \operatorname{Li}_2\left(\frac{l + E - m}{2l}\right) \Big] + \frac{1}{4l} \left[3E + m - E \ln\left(\frac{2E + 2m}{m}\right) \right] \ln\left(\frac{E + l}{E - l}\right), \quad (4)$$

$$V_2(E) = \frac{m}{4l} \ln\left(\frac{E + l}{E - l}\right). \quad (5)$$

λ is a small photon mass introduced to regularize the infrared divergence and $l = \sqrt{E^2 - m^2}$ is the three-momentum of the electron. The dilogarithm $\operatorname{Li}_2(x)$ is defined by

$$\operatorname{Li}_2(x) = - \int_0^x dt \frac{\ln(1 - t)}{t}.$$

The L part of eq. (2) (with $g_L = 1$) is identical to the formula for the one-loop photonic corrections to the $\nu_e + e \rightarrow \nu_e + e$ differential cross section computed long ago in the pioneering work of Lee and Sirlin [2] using the effective four-fermion Fermi V–A Lagrangian. The analogous formula for the reaction involving an anti-neutrino $\bar{\nu}_e$ (rather than a neutrino ν_e) can be found in the same article and coincides¹ with the R part of eq. (2) (with $g_R = 1$). This identity is simply due to the fact that the cross section for antineutrinos in the local V–A theory is the same as that for neutrinos calculated with a V+A coupling. On the contrary, the LR part of eq. (2) has clearly no analogue in the V±A theory, but can be derived very easily once the L and R parts are known.

The $\nu_e + e \rightarrow \nu_e + e + \gamma$ differential cross section with emission of a soft photon was computed in ref. [2], once again by using the effective four-fermion Fermi V–A Lagrangian. It can be identified with the L part (with $g_L = 1$) of the soft photon corrections to the tree level result in eq. (1). The L , R and LR parts of these corrections (with $g_L = g_R = 1$) are however identical, because the whole soft bremsstrahlung cross section is proportional to its lowest-order elastic prediction. We can therefore write the soft photon emission cross section in the following factorized form:

$$\left[\frac{d\sigma}{dE} \right]_{\text{SB}} = \frac{\alpha}{\pi} I_\gamma(E, \epsilon) \left[\frac{d\sigma}{dE} \right]_0, \quad (6)$$

with

$$\begin{aligned} I_\gamma(E, \epsilon) = & \left(2 \ln \frac{\lambda}{\epsilon} \right) \left[1 - \frac{E}{2l} \ln\left(\frac{E + l}{E - l}\right) \right] + \frac{E}{2l} \left\{ L\left(\frac{E + l}{E - l}\right) - L\left(\frac{E - l}{E + l}\right) \right. \\ & \left. + \ln\left(\frac{E + l}{E - l}\right) \left[1 - 2 \ln\left(\frac{l}{m}\right) \right] \right\} + 1 - 2 \ln 2 \end{aligned} \quad (7)$$

and

$$L(x) = \int_0^x dt \frac{\ln|1 - t|}{t}.$$

¹with the exception of a minor typographical error in their eq. 22, where the square bracket multiplying I_{rad} should be squared. We thank Alberto Sirlin for confirming this point.

(For $x \in \mathbb{R}$, $L(x) = -\text{Re}[\text{Li}_2(x)]$.) This result is valid under the assumption that ϵ , the maximum soft photon energy, is much smaller than m or the final momentum of the electron. As we mentioned earlier, in the following we will discuss numerical results for the case in which ϵ is an arbitrary parameter. The reader will notice that the sum $(V_1(E) + I_\gamma(E, \epsilon))$ does not depend on λ , the infrared regulator. Indeed, the infrared divergence of the virtual corrections (eq. (2)) is canceled by that arising from the soft photon emission (eq. (6)).

The SM prediction for the differential neutrino electron cross section

$$\nu_l + e \rightarrow \nu_l + e (+\gamma), \quad (8)$$

where $(+\gamma)$ indicates the possible emission of a photon, can be cast, up to corrections of $O(\alpha)$, in the following form:

$$\begin{aligned} \left[\frac{d\sigma}{dE} \right]_{\text{SM}} = & \frac{2mG_\mu^2}{\pi} \left\{ g_L^2(E) \left[1 + \frac{\alpha}{\pi} f_L(E, \nu) \right] + g_R^2(E) (1-z)^2 \left[1 + \frac{\alpha}{\pi} f_R(E, \nu) \right] \right. \\ & \left. - g_L(E) g_R(E) \left(\frac{mz}{\nu} \right) \left[1 + \frac{\alpha}{\pi} f_{LR}(E, \nu) \right] \right\}. \end{aligned} \quad (9)$$

(We remind the reader that terms of $O(q^2/M_W^2)$ are neglected in our analysis.) The deviations of the functions $g_L(E)$ and $g_R(E)$ from the lowest-order values g_L and g_R reflect the effect of the electroweak corrections, which have been studied by several authors [6, 8, 13]. (See ref. [13] for simple numerical results.)

The functions $f_X(E, \nu)$ ($X = L, R$ or LR) describe the QED effects (real and virtual photons). For simplicity of notation their ν dependence will be dropped in the following. Each of these functions is the sum of virtual (V), soft (SB) and hard (HB) corrections,

$$f_X(E) = f_X^V(E) + f_X^{SB}(E, \epsilon) + f_X^{HB}(E, \epsilon). \quad (10)$$

The analytic expressions for $f_X^V(E)$ and $f_X^{SB}(E, \epsilon)$ can be immediately read from eqs. (2) and (6) respectively (the latter being valid only in the small ϵ limit) and their sums, which are infrared-finite, will be denoted by

$$f_X^{VS}(E, \epsilon) = f_X^V(E) + f_X^{SB}(E, \epsilon). \quad (11)$$

Analytic expressions from which one can obtain $f_L^{HB}(E, \epsilon)$ and $f_R^{HB}(E, \epsilon)$ were calculated long ago by Ram in the small ϵ approximation, keeping the logarithmically divergent terms proportional to $\ln(\epsilon/m)$ but neglecting the remaining ϵ -dependent terms [3]. The formulae are nonetheless long and complicated. The function $f_{LR}^{HB}(E, \epsilon)$ was computed only very recently [14].

In ref. [14] we created **BC**, a combined **Mathematica**-**FORTTRAN** code² to compute the $f_X^{HB}(E, \epsilon)$ functions for arbitrary positive values of the parameter ϵ up to the kinematic

²The code **BC**, available upon request, computes all QED corrections discussed in this note.

limit ν (ν , the incident neutrino energy in the laboratory system, is also the maximum possible energy of the emitted photon). We refer the interested reader to this paper for details concerning this computation and the comparison with Ram's results. Our results confirm Ram's ones in the small ϵ limit. If ϵ is not small, the discrepancy between them can become very large. Moreover, Ram's results for $f_L^{HB}(E, \epsilon)$ and $f_R^{HB}(E, \epsilon)$ are not always positive. This is of course an unphysical property because the HB differential cross section, being a transition probability for a physical process, cannot be negative. Our functions $f_L^{HB}(E, \epsilon)$ and $f_R^{HB}(E, \epsilon)$ are always positive (or zero).

The total $O(\alpha)$ QED corrections $f_L(E)$ and $f_R(E)$, given by the sum of V, SB and HB contributions (see eq. (10)), can be easily obtained by adding the analytic results of eqs. (2) and (6) to Ram's HB (lengthy) ones. Both SB and HB corrections were computed in the small ϵ approximation, and the logarithmically divergent terms proportional to $\ln(\epsilon/m)$ exactly drop out upon adding these soft and hard contributions. The remaining ϵ -dependent terms, which were neglected in both SB and HB calculations, must cancel in the sum as well, and are therefore irrelevant in the computation of the total QED corrections of eq. (9). The LR case is slightly different: Ram's formulae, which were used to derive the small ϵ approximation for $f_L^{HB}(E, \epsilon)$ and $f_R^{HB}(E, \epsilon)$, do not provide us with the corresponding LR correction. In order to compute $f_{LR}(E)$ we have therefore added the V and SB analytic results of eqs. (2) and (6) to our HB numerical results. The "exact" ϵ dependence of our HB results is not completely canceled by that of the SB, which includes only terms proportional to $\ln(\epsilon/m)$, and the sum $f_{LR}(E)$ contains therefore a residual (not logarithmically divergent) dependence on the photon energy threshold ϵ . This spurious dependence has been minimized by fixing ϵ to be a very small value ϵ_{LR} chosen so as to have an estimated induced relative error as small as $O(0.1\%)$ ³.

Eqs. (2) and (6) determine the analytic expression of $f_X^{VS}(E, \epsilon)$ (the infrared-finite sum of V and SB corrections) in the small ϵ approximation. But the complete ϵ dependence of our numerical $f_X^{HB}(E, \epsilon)$ computations, combined with the knowledge of the above described $f_X(E)$ functions, allows us to determine also the "exact" $f_X^{VS}(E, \epsilon)$ functions via the subtraction

$$f_X^{VS}(E, \epsilon) = f_X(E) - f_X^{HB}(E, \epsilon). \quad (12)$$

These will be the "exact" VS corrections employed in the rest of our analysis.

In fig. 1 we plotted the functions $f_X(E)$ (thick solid), $f_X^{HB}(E, \epsilon)$ (medium solid) and $f_X^{VS}(E, \epsilon)$ (thin solid) for $\nu = 0.862$ MeV. The threshold ϵ in the VS and HB functions was set to 0.02 MeV. In fig. 2 we plotted the same functions with $\nu =$

³with the exception of E belonging to a tiny interval of $O(\epsilon_{LR})$ at the endpoint E_{\max} . Note that 0.1% is also the relative numerical uncertainty used by our code BC in the computation of the functions $f_X^{HB}(E, \epsilon)$ and produces a totally negligible relative error $(\alpha/\pi)f_X^{HB}(E, \epsilon) \times 0.1\%$ in the corresponding X parts of the differential cross section in eq. (9).

10 MeV and $\epsilon = 1$ MeV. These two values of the neutrino energy were chosen for their relevance in the study of solar neutrinos: $\nu = 0.862$ MeV is the energy of the (almost) monochromatic neutrinos produced by electron capture on ${}^7\text{Be}$ in the solar interior, while $\nu = 10$ MeV belongs to the continuous energy spectrum of the solar neutrinos that originate from the decay of ${}^8\text{B}$. In figs. 1 and 2 we also plotted the simple approximate formulae for $f_x(E)$ introduced in ref. [13] (dotted lines). These compact analytic expressions were obtained by modifying the expressions of ref. [8], which had been evaluated in the extreme relativistic approximation. (The LR term of the differential cross section, being proportional to (m/ν) , vanishes in the extreme relativistic limit and, therefore, cannot be derived from ref. [8]. As a consequence, the LR approximation of ref. [13] is only a (very educated!) guess.) Thanks to their simplicity, the compact formulae of ref. [13] are easy to use and are employed, for example, by the Super-Kamiokande collaboration in their Monte Carlo simulations for the analysis of the solar neutrino energy spectrum.

As it was noted in refs. [3, 13], all $f_x(E)$ functions contain a term which diverges logarithmically at the end of the spectrum. This feature, related to the infrared divergence, is similar to the one encountered in the QED corrections to the μ -decay spectrum [25, 26]. If E gets very close to the endpoint we have $(\alpha/\pi)f_x(E) \approx -1$, clearly indicating a breakdown of the perturbative expansion and the need to consider multiple-photon emission. However, this divergence can be easily removed (in agreement with the KLN theorem [26, 27]) by integrating the differential cross section over small energy intervals corresponding to the experimental energy resolution. We also note that the singularity of $f_{LR}(E)$ for $E = m$ does not pose a problem, as the product $(mz/\nu)f_{LR}(E)$, which appears in the LR part of the differential cross section, is finite in the limit $E \rightarrow m$. This can be seen from the dashed line in the LR plot of fig. 1, which indicates the function $(mz/\nu)f_{LR}(E)$. In the same plot, the dot-dashed line is the product of the $f_{LR}(E)$ approximation of ref. [13] and (mz/ν) .

3 Spectrum of the Combined Energy of Electron and Photon

We will now turn our attention to the analysis of the differential $\nu_l + e \rightarrow \nu_l + e (+\gamma)$ cross section relevant to experiments measuring the *total combined energy* of the recoil electron and a possible accompanying photon emitted in the scattering process. We will begin by considering bremsstrahlung events with a photon of energy ω larger than the usual threshold ϵ (HB).

The HB differential cross section with respect to the sum of the electron and photon energies is computed by our code BC (see ref. [14]). The code first calculates the HB corrections to the energy spectrum of the final neutrino; the HB differential cross section $[d\sigma/d(E+\omega)]_{\text{HB}}$ is then immediately derived via energy conservation. A

check of the consistency of our results was performed by comparing the values of the total HB cross section $\sigma_{\text{HB}}(\nu, \epsilon)$ obtained by integrating both $[d\sigma/d(E + \omega)]_{\text{HB}}$ and the differential HB cross section of sect. 2 for several values of ν and ϵ . All relative deviations were found to be smaller than 0.1% (which is also the relative accuracy of the integrands). In the elastic reaction $\nu_l + e \rightarrow \nu_l + e$, the final neutrino energy ν' ranges from $\nu'_{\text{min}} = \nu m/(2\nu + m)$ to $\nu'_{\text{max}} = \nu$ (the value $\nu' = \nu'_{\text{min}}$ occurs when the final electron and neutrino are scattered back to back, with the electron moving in the forward direction with $E = E_{\text{max}}$; the value $\nu' = \nu'_{\text{max}}$ occurs in the forward scattering situation). When a photon of energy $\omega > \epsilon$ is emitted, ν' varies between 0 and $\nu - \epsilon$, while the variable $E + \omega$ varies between $m + \epsilon$ and $m + \nu$ (note that $m + \nu = E_{\text{max}} + \nu'_{\text{min}}$).

How do we combine virtual, soft and hard bremsstrahlung contributions in order to evaluate the complete $O(\alpha)$ QED prediction for the differential cross section $d\sigma/d(E + \omega)$ of reaction (8)? In sect. 2 we computed the total QED corrections by simply adding these three parts. Their sum does not depend on the threshold ϵ . The combination of VS and HB terms requires here a more careful treatment for which we refer the interested reader, once again, to ref. [14]. We will only discuss the results of this analysis, which can be summarized in the following simple way. Let's consider an experimental setup for ν - e scattering able to measure the photon energy if it's higher than a threshold ϵ , but completely blind to low energy photons ($\omega < \epsilon$). Let's also assume that the electron energy E is precisely measurable independently of its value. This detector can measure the usual electron spectrum $d\sigma/dE$ as well as the differential cross section $d\sigma/dE_\omega$, where the variable E_ω is defined as follows,

$$E_\omega \equiv \begin{cases} E + \omega & \text{if } \omega \geq \epsilon \\ E & \text{if } \omega < \epsilon \end{cases} . \quad (13)$$

The SM prediction for the spectrum of the combined energy of electron and photon in reaction (8) can be cast, up to corrections of $O(\alpha)$, in the form

$$\begin{aligned} \left[\frac{d\sigma}{dE_\omega} \right]_{\text{SM}} &= \frac{2mG_\mu^2}{\pi} \left\{ g_L^2(E_\omega) \left[\theta + \frac{\alpha}{\pi} \bar{f}_L(E_\omega, \epsilon, \nu) \right] + g_R^2(E_\omega) (1 - z_\omega)^2 \left[\theta + \frac{\alpha}{\pi} \bar{f}_R(E_\omega, \epsilon, \nu) \right] \right. \\ &\quad \left. - g_L(E_\omega) g_R(E_\omega) \left(\frac{m z_\omega}{\nu} \right) \left[\theta + \frac{\alpha}{\pi} \bar{f}_{LR}(E_\omega, \epsilon, \nu) \right] \right\}, \end{aligned} \quad (14)$$

where $z_\omega = (E_\omega - m)/\nu$ and $\theta = \theta(E_{\text{max}} - E_\omega)$. As we mentioned in sect. 2, the deviations of the functions $g_L(E_\omega)$ and $g_R(E_\omega)$ from the lowest-order values g_L and g_R reflect the effect of the electroweak corrections (for virtual corrections it is $\omega = 0$ and $E_\omega = E$). The functions $\bar{f}_X(E_\omega, \epsilon, \nu)$ ($X = L, R$ or LR), defined in the range $[m, m + \nu]$, describe the QED effects and can be written in the very simple form (once again, for simplicity of notation, we will drop their ν dependence)

$$\bar{f}_X(E_\omega, \epsilon) = f_X^{\text{VS}}(E_\omega, \epsilon) + \bar{f}_X^{\text{HB}}(E_\omega, \epsilon), \quad (15)$$

where $f_X^{VS}(E_\omega, \epsilon)$ are the “exact” VS corrections of sect. 2 (eq. (12)) and the functions $\bar{f}_X^{HB}(E_\omega, \epsilon)$ are derived by dividing the L , R and LR parts of the above-mentioned HB cross section $[d\sigma/d(E+\omega)]_{\text{HB}}$ by Cg_L^2 , $Cg_R^2(1-z_\omega)^2$ and $-Cg_Lg_R(mz_\omega/\nu)$ respectively, with $C = 2mG_\mu^2\alpha/\pi^2$. The θ functions in eq. (14) reflect the fact that the lowest order prediction for $d\sigma/dE_\omega$ has a step at $E_\omega = E_{\text{max}}$ and is zero if E_ω lies outside the elastic range $[m, E_{\text{max}}]$. The VS functions $f_X^{VS}(E_\omega, \epsilon)$ are proportional to the same θ function, while the corrections $\bar{f}_X^{HB}(E_\omega, \epsilon)$ are set to zero if $E_\omega \notin [m + \epsilon, m + \nu]$. We would like to emphasize the ϵ dependence of the complete QED corrections $\bar{f}_X(E_\omega, \epsilon)$, to be contrasted with the ϵ independence of the $f_X(E)$ functions of sect. 2.

In figs. 3 and 4 we compare the results of sects. 2 and 3. In fig. 3 we chose $\nu = 0.862$ MeV and plotted the functions $f_X(E)$ (thick) and $\bar{f}_X(E_\omega, \epsilon)$ for $\epsilon = 0.1$ MeV (medium) and 0.001 MeV (thin). In fig. 4 we plotted the same functions with $\nu = 10$ MeV ($\epsilon = 1$ MeV, 0.1 MeV).

We would like to remind the reader that the functions $f_X(E)$ can be obtained from $\bar{f}_X(E_\omega, \epsilon)$ by simply setting $\epsilon = \nu$. The limiting case $\epsilon = 0$ was studied in detail in ref. [10] (in particular, the results of the second article of this reference were obtained, like ours, without employing the ultrarelativistic approximation $E \gg m$).

4 Discussion and Conclusions

When are the results of sects. 2 and 3 applicable? In sect. 2 we presented the $O(\alpha)$ SM prediction for the electron spectrum in the reaction $\nu_l + e \rightarrow \nu_l + e (+\gamma)$ (eq. (9)), where $(+\gamma)$ indicates the possible emission of a photon. In this calculation we assumed that the final-state photon is not detected and, as a consequence, we integrated over all possible values of the photon energy ω . Therefore, eq. (9) is the appropriate theoretical prediction to use in the analysis of ν - e scattering when the detector is completely blind to photons of all energies, but can precisely measure E , the energy of the electron. Of course, a detector could provide more information by detecting photons as soon as their energy is above an experimental threshold ϵ . In this case, still assuming a precise determination of E , one can employ eq. (9), minus its HB correction, to analyze those events which are counted as nonradiative (elastic), while the HB part can be used, at least in principle, for a separate determination of the inelastic cross section. Indeed, contrary to previous calculations, our predictions are valid for an arbitrary value of the threshold ϵ (and include the previously unknown LR term).

In sect. 3 we examined the spectrum of the total combined energy of the recoil electron and a possible accompanying photon emitted in the scattering process (eq. (14)). This type of analysis is useful when the photon energy ω cannot be separately determined although it fully contributes to the precise total energy measurement if its value is above a specific threshold ϵ . Let’s consider an experimental setup able to

measure the photon energy if it's higher than ϵ , but completely blind to low energy photons ($\omega < \epsilon$). Let's also assume that the electron energy E is precisely measurable independently of its value. This detector can determine both the differential cross section $d\sigma/dE_\omega$ (eq. (14)) and the electron spectrum $d\sigma/dE$ (eq. (9)) (as well as its separate HB component). There are experiments, however, which cannot measure E , but only E_ω , with a specific value of the threshold ϵ . BOREXINO [28] and KamLAND [29], for example, are liquid scintillation detectors in which photons and electrons induce practically the same response. If a photon is emitted in the ν - e scattering process, its energy ω is counted together with E , provided their sum lies within a specific range. The appropriate theoretical prediction for their analysis is given, therefore, by the cross section $d\sigma/dE_\omega$ of eq. (14) with a very small value of ϵ (for the case $\nu = 0.862$ MeV see the thin lines in fig. 3). However, we should point out that although the QED corrections $f_X(E)$ (in eq. (9)) and $\bar{f}_X(E_\omega, \epsilon)$ with small ϵ (in eq. (14)) are different, their numerical values are very small when $\nu = 0.862$ MeV, the energy of the monochromatic neutrinos produced by electron capture on ${}^7\text{Be}$ in the solar interior. In fact, as shown in fig. 3, both $(\alpha/\pi)f_X(E)$ and $(\alpha/\pi)\bar{f}_X(E_\omega, \epsilon)$ with small ϵ are in this case of $O(\lesssim 1\%)$, and neither of the above collaborations is likely to reach this high level of accuracy in their analyses of the crucial ${}^7\text{Be}$ line.

There are detectors in which it might not be possible to identify the measured energy with either E or E_ω . Indeed, the electron and the photon may produce indistinguishable signals and the total observed energy might not be the simple sum of E and ω , but some other function of these two variables. Super-Kamiokande (SK), for example, a water Cherenkov counter measuring the light emitted by electrons recoiling from neutrino scattering, uses the number of hit photomultiplier tubes to determine the electron energy. However, a photon emitted in the scattering process may induce additional hits indistinguishable from those of the electron. Moreover, a photon and an electron of the same energy may produce different numbers of hits and, therefore, it might not be possible to identify the total measured energy with the sum $E + \omega$.

SK measures solar neutrinos with energies varying from 5 to 18 MeV. For $\nu = 10$ MeV, fig. 4 shows that the QED corrections to the differential cross sections $d\sigma/dE$ (eq. (9)) and $d\sigma/dE_\omega$ (eq. (14)) are of $O(1\%)$. Corrections of this order may be relevant for the analysis of the very precise data obtained by this collaboration. In fact, SK's Monte Carlo simulations of the expected energy spectrum of recoil electrons from solar neutrino scattering include the QED corrections of ref. [13] (as well as the EW ones). As we investigated in sect. 2, these corrections provide good approximations of the complete $O(\alpha)$ QED corrections $f_X(E)$ to the electron spectrum of eq. (9) (see fig. 2). Our previous discussion, however, seems to suggest that these corrections are not appropriate for SK's solar neutrino analysis. On the other hand, the SM prediction for the spectrum of the combined energy of electron and photon of sect. 3 (eq. (14)) may be suitable, but only if we can assume a similar efficiency in the detection of photons and electrons, and if also relatively low energy electrons contribute to the

total energy measurement. If these conditions are not met, and the precision of the data requires it, one should probably perform a dedicated analysis of the double differential cross section $d^2\sigma/(dE d\omega)$ with a response function specifically designed for this detector. A triple differential cross section $d^3\sigma/(dE d\omega d\phi)$, where ϕ is the angle between the directions of the electron and the photon, may also be useful (see the first article of ref. [11]).

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References

- [1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109** (1958) 193.
- [2] T. D. Lee and A. Sirlin, Rev. Mod. Phys. **36** (1964) 666.
- [3] M. Ram, Phys. Rev. **155** (1967) 1539.
- [4] G. 't Hooft, Phys. Lett. **B37** (1971) 195.
- [5] E. D. Zhizhin, R. V. Konoplich, Yu. P. Nikitin, and B. U. Rodionov, JETP Lett. **19** (1974) 36; E. D. Zhizhin, R. V. Konoplich and Y. P. Nikitin, Sov. Phys. J. **18** (1975) 1709 (No.12), translated from Izv. Vuz, Fiz. (1975) No.12 82-89; *Elementary particles and cosmic rays*, Atomizdat, Moscow, (1976), 57-71 (in russian).
- [6] P. Salomonson and Y. Ueda, Phys. Rev. **D11** (1975) 2606; M. Green and M. Veltman, Nucl. Phys. **B169** (1980) 137; Erratum-ibid. **B175** (1980) 547; W. J. Marciano and A. Sirlin, Phys. Rev. **D22** (1980) 2695; K. Aoki, Z. Hioki, R. Kawabe, M. Konuma and T. Muta, Prog. Theor. Phys. **65** (1981) 1001.
- [7] K. Aoki and Z. Hioki, Prog. Theor. Phys. **66** (1981) 2234; Z. Hioki, Prog. Theor. Phys. **67** (1982) 1165.
- [8] S. Sarantakos, A. Sirlin and W.J. Marciano, Nucl. Phys. **B217** (1983) 84.
- [9] D. Y. Bardin and V. A. Dokuchaeva, Sov. J. Nucl. Phys. **39** (1984) 563.
- [10] D. Y. Bardin and V. A. Dokuchaeva, Nucl. Phys. **B246** (1984) 221; Sov. J. Nucl. Phys. **43** (1986) 975.
- [11] J. Bernabeu, S. M. Bilenky, F. J. Botella and J. Segura, Nucl. Phys. **B426** (1994) 434; J. Bernabeu, L. G. Cabral-Rosetti, J. Papavassiliou and J. Vidal, Phys. Rev. **D62** (2000) 113012.

- [12] J. N. Bahcall, Rev. Mod. Phys. **59** (1987) 505.
- [13] J. N. Bahcall, M. Kamionkowski and A. Sirlin, Phys. Rev. **D51** (1995) 6146.
- [14] M. Passera, BUTP 2000/11, hep-ph/0011190.
- [15] Kamiokande Collaboration, K.S. Hirata *et al.*, Phys. Rev. Lett. **63** (1989) 16.
- [16] J. N. Bahcall, Phys. Rept. **333** (2000) 47; these proceedings.
- [17] J. N. Bahcall, Phys. Rev. **D44** (1991) 1644.
- [18] J. N. Bahcall, P. I. Krastev and E. Lisi, Phys. Rev. **C55** (1997) 494; J. N. Bahcall and P. I. Krastev, Phys. Rev. **C56** (1997) 2839.
- [19] E. Lisi and D. Montanino, Phys. Rev. **D56** (1997) 1792; G. L. Fogli, E. Lisi and D. Montanino, Phys. Lett. **B434** (1998) 333.
- [20] A. de Gouvea and H. Murayama, Phys. Rev. Lett. **82** (1999) 3392; JHEP**0008** (2000) 025.
- [21] M. C. Gonzalez-Garcia and C. Pena-Garay, talk presented at Neutrino 2000, Sudbury, Canada, June 16-21, 2000, hep-ph/0009041; M. C. Gonzalez-Garcia, M. Maltoni, C. Pena-Garay and J. W. Valle, Phys. Rev. **D63** (2001) 033005.
- [22] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81** (1998) 1158; Erratum-*ibid.* **81** (1998) 4279; Phys. Rev. Lett. **82** (1999) 2430; Y. Suzuki and Y. Totsuka (Eds.), Neutrino 98, Proceedings of the XVIII International Conference on Neutrino Physics and Astrophysics, Takayama, Japan, June 4-9, 1998, Nucl. Phys. Proc. Suppl. **77** (1999); Y. Suzuki, talk presented at Neutrino 2000, Sudbury, Canada, June 16-21, 2000.
- [23] A. Sirlin, Rev. Mod. Phys. **50** (1978) 573; Phys. Rev. **D22** (1980) 971.
- [24] A. Ferroglia, G. Ossola and A. Sirlin, Nucl. Phys. **B560** (1999) 23.
- [25] R. E. Behrends, R. J. Finkelstein and A. Sirlin, Phys. Rev. **101** (1956) 866.
- [26] T. Kinoshita and A. Sirlin, Phys. Rev. **113** (1959) 1652.
- [27] T. Kinoshita, J. Math. Phys. **3** (1962) 650; T. D. Lee and M. Nauenberg, Phys. Rev. **133** (1964) B1549.
- [28] G. Ranucci, talk presented at Neutrino 2000, Sudbury, Canada, June 16-21, 2000.
- [29] A. Piepke, talk presented at Neutrino 2000, Sudbury, Canada, June 16-21, 2000.

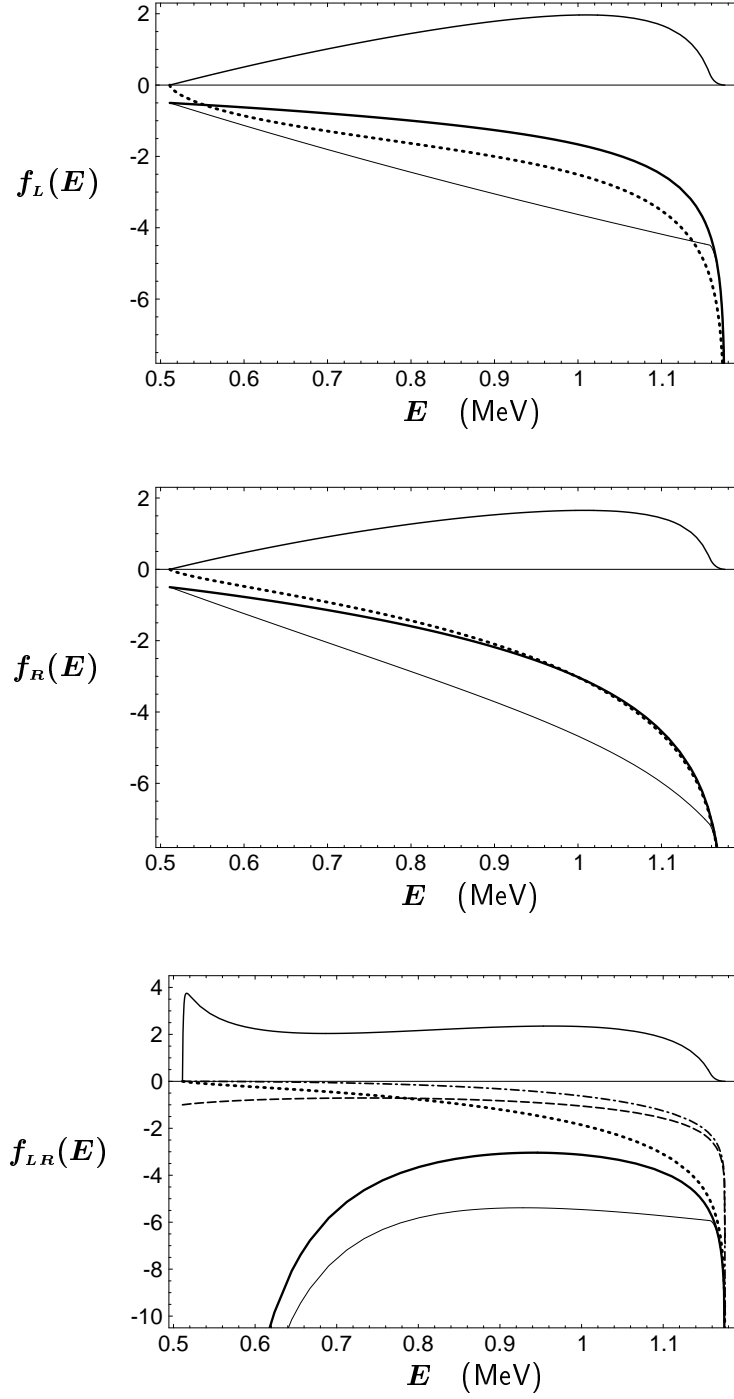


Figure 1: The functions $f_x(E)$ (thick solid), $f_x^{HB}(E, \epsilon)$ (medium solid) and $f_x^{VS}(E, \epsilon)$ (thin solid) for $\nu = 0.862$ MeV and $\epsilon = 0.02$ MeV. The dotted lines represent the $f_x(E)$ approximations of ref. [13]. In the LR figure, the dot-dashed line is the product of the $f_{LR}(E)$ approximation of ref. [13] and (mz/ν) , while the dashed line indicates the product $(mz/\nu)f_{LR}(E)$.

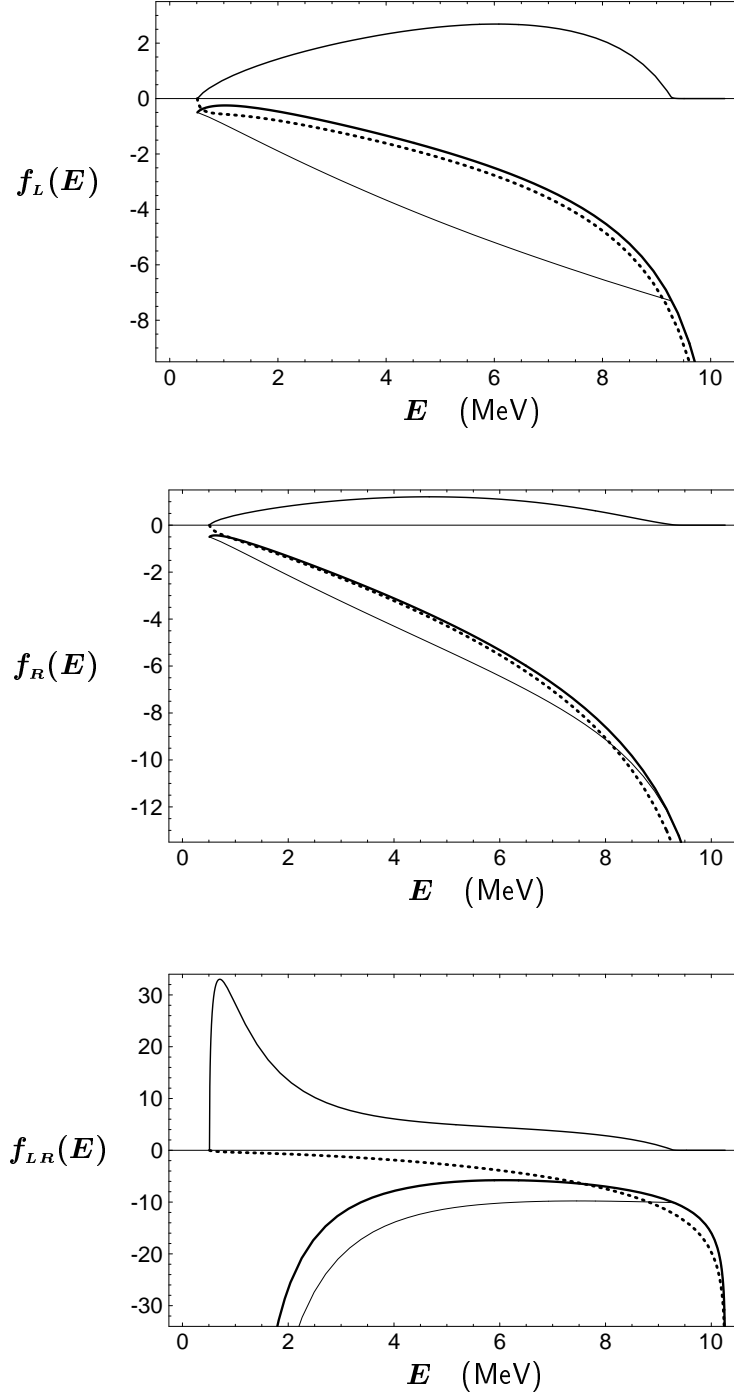


Figure 2: Same as Fig. 1, but for $\nu = 10$ MeV and $\epsilon = 1$ MeV. The dashed and dot-dashed lines are very close to zero and are not indicated.

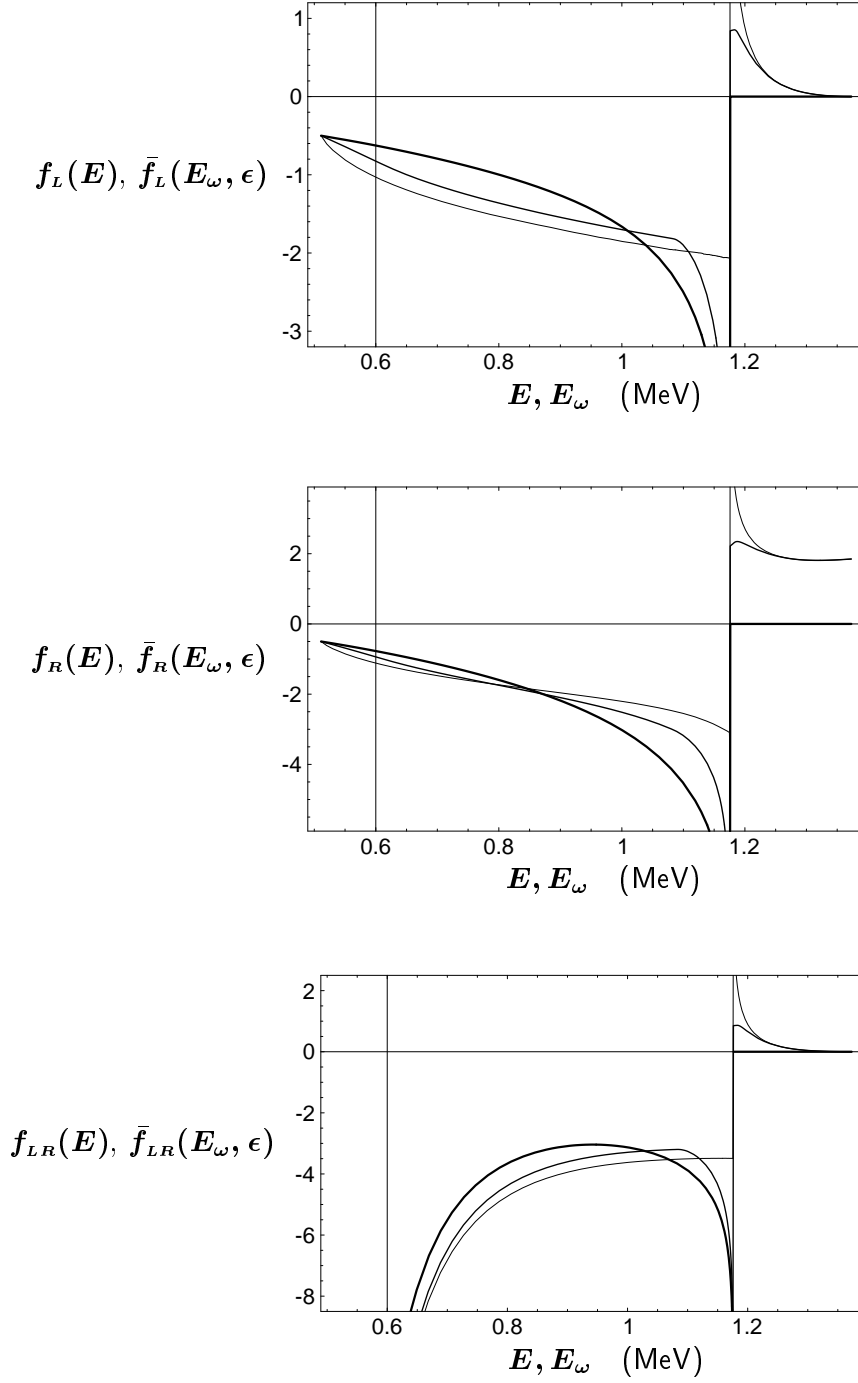


Figure 3: The functions $f_x(E)$ (thick) and $\bar{f}_x(E_\omega, \epsilon)$ for $\epsilon = 0.1$ MeV (medium) and 0.001 MeV (thin). $\nu = 0.862$ MeV.

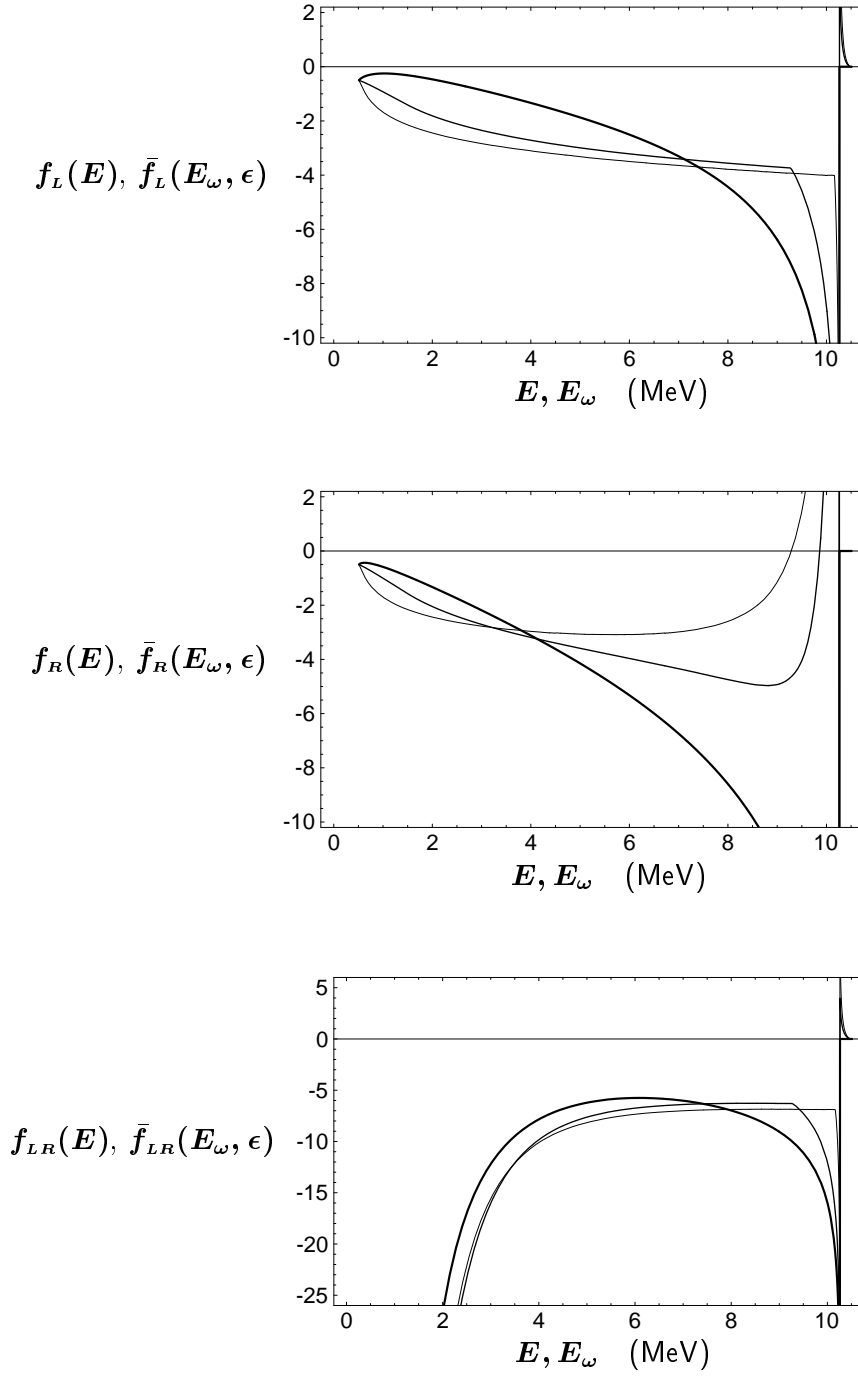


Figure 4: The functions $f_x(E)$ (thick) and $\bar{f}_x(E_\omega, \epsilon)$ for $\epsilon = 1$ MeV (medium) and 0.1 MeV (thin). $\nu = 10$ MeV.